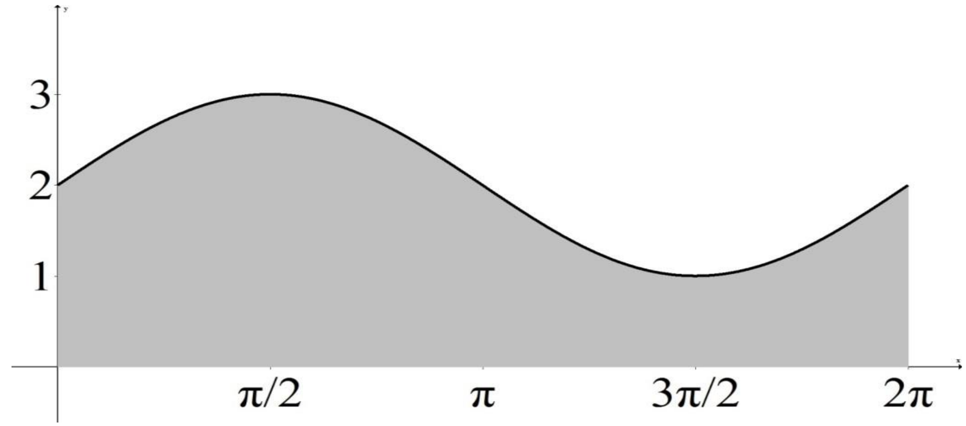
Closing today at 11pm:HW_1A,1B,1C
Closing next Wed: HW_2A,2B
Closing next Fri: HW_2C
(Note: No class Monday and no MSC)
Note on quick bounds (HW_1C: 9,10)

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

Example: Consider the area under

f(x) = sin(x) + 2

on the interval $x = 0$ to $x = 2\pi$.



(a) What is the max of f(x)? (label M)
(b) What is the min of f(x)? (label m)
(c) Draw **one** rectangle that contains all the shaded area? What can you conclude?
(d) Draw **one** rectangle that is completely inside the shaded area? Conclusion?

5.3 The Fundamental Theorem of Calculus (FTOC)

Motivational Task:

Consider the function *f(t) = 3t*. Draw the graph and using area formulas you know, compute:

1. $\int_0^1 f(t)dt$
2. $\int_0^{10} f(t)dt$
3. $g(x) = \int_0^x f(t)dt$

Any observations?

Fundamental Theorem of Calculus (Part 1):

Areas under graphs are antiderivatives!

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

In other words, for any constant a, the "accumulated signed area" formula

$$F(x) = \int_a^x f(t)dt$$

is an antiderivative of $f(x)$.

Motivational Task:

Again, consider the function *f(t) = 3t*.

Using the area of the triangle again, simplify, then differentiate:

$$1.\ h(x) = \int_{0}^{1+x^3} f(t)dt\ , h'(x) = ?$$

$$2.\ k(x) = \int_{x^2}^{1+x^3} f(t)dt\ , k'(x) = ?$$

Any observations?

General form of FTOC (Part 1):

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)dt\right) = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$$

Fundamental Theorem of Calculus (Part 2):

If F(x) <u>any</u> antiderivative of f(x),

$$\int_{a}^{b} f(x)dx = F(b) - F(a)$$